



# Maestría en Ingeniería en Automatización de Procesos Industriales

Position control of five degrees of  
freedom robotic arm

*Miguel Angel Vivanco Bautista*

*Aldo Hernández Díaz*

*Luis Hernández Martínez*



*Symposium  
de Posgrado*

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Vivanco Bautista M.A., Hernández Díaz A. and Hernández Martínez L.  
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mvivanco4475@uppuebla.edu.mx

Tercer Carril del Ejido Serrano S/N, San Mateo Cuanalá, Juan C. Bonilla, Puebla, México

## 1. Introduction

Everyday, the importance of robotic arms is increasing in industry and medical applications due to its accurately and stability for high speed.

One of the most important challenges in programming arms is the control positioning of the end effector. It is very important to develop applications, i.e.:

- Handling and transfer chemical components remotely and safely, that could put on risk the operator in the chemical industry.
- In the automotive area, sometimes accuracy of assembling tends to be less than 1 mm which requires high accuracy.
- The medical area, particularly with remote surgery or remote with real-time monitoring, any sudden movement of the manipulator increases the risk of damaging an organ in the patient.

The design of position control is a non-trivial task, it requires the formulation of a mathematical model consisting of the kinematic and dynamic models. In this master thesis, we develop the position control of five degrees of freedom robotic arm. Nowadays, mathematical model of prototype robotic arm has been developed, evaluated and validated by Matlab simulations. Also, its physical implementation, to carried out using STM32 microcontroller, that reads and write the current joint variable values and set desired joint position.

## 2. Objectives

### 2.1. General objective

Implementation of a PD control for a manipulator arm of five degrees of freedom

### 2.2. Specific objectives

- Get the theoretical models of the kinematics and dynamics of the manipulator arm of five degrees of freedom.
- Simulate the mathematical model and implement it in the manipulator.
- Implement and evaluate the accuracy of PD position control to the manipulator

## 3. Methods

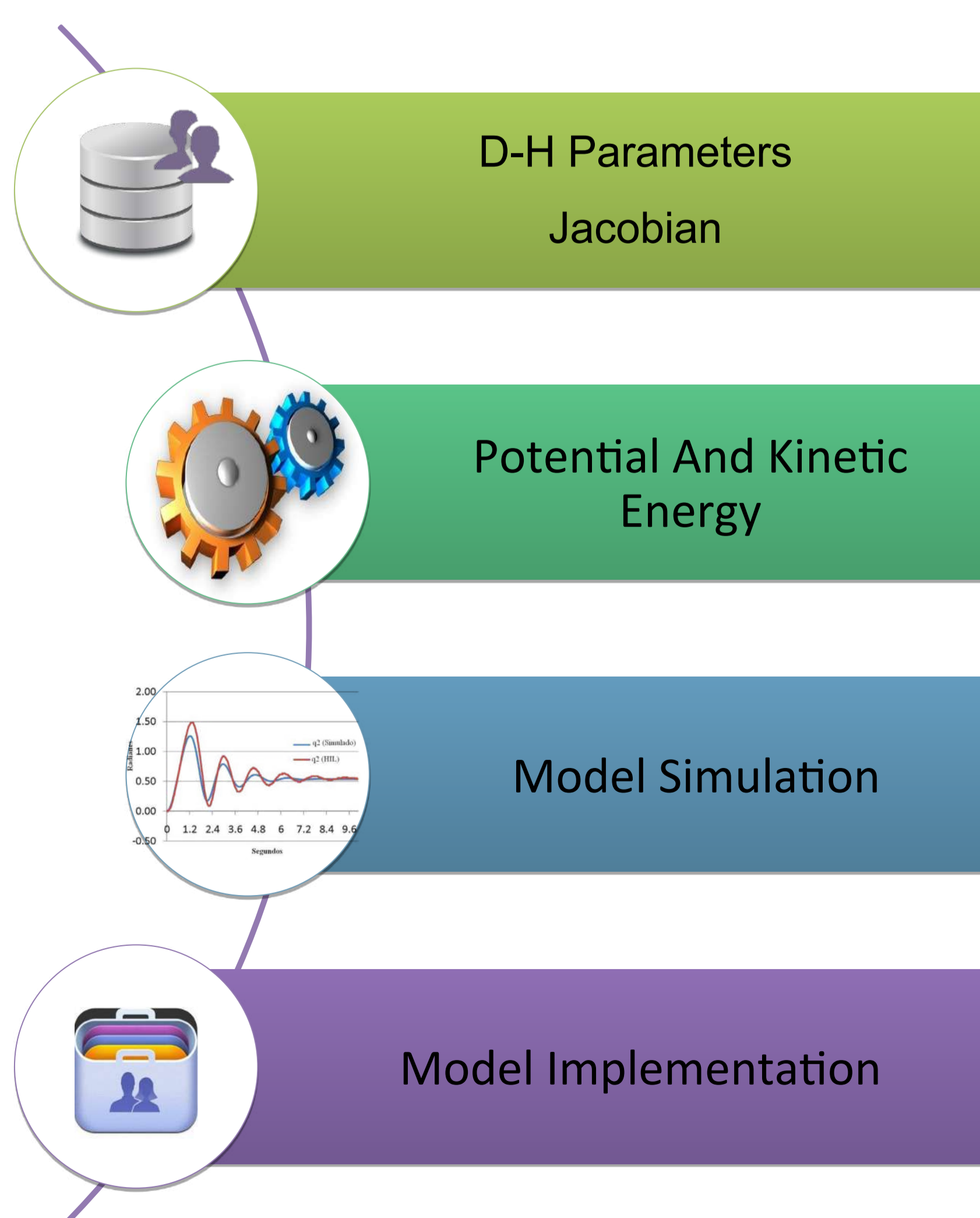


Figure 1. Methodology used in this research.

### 3.1 Manipulator configuration

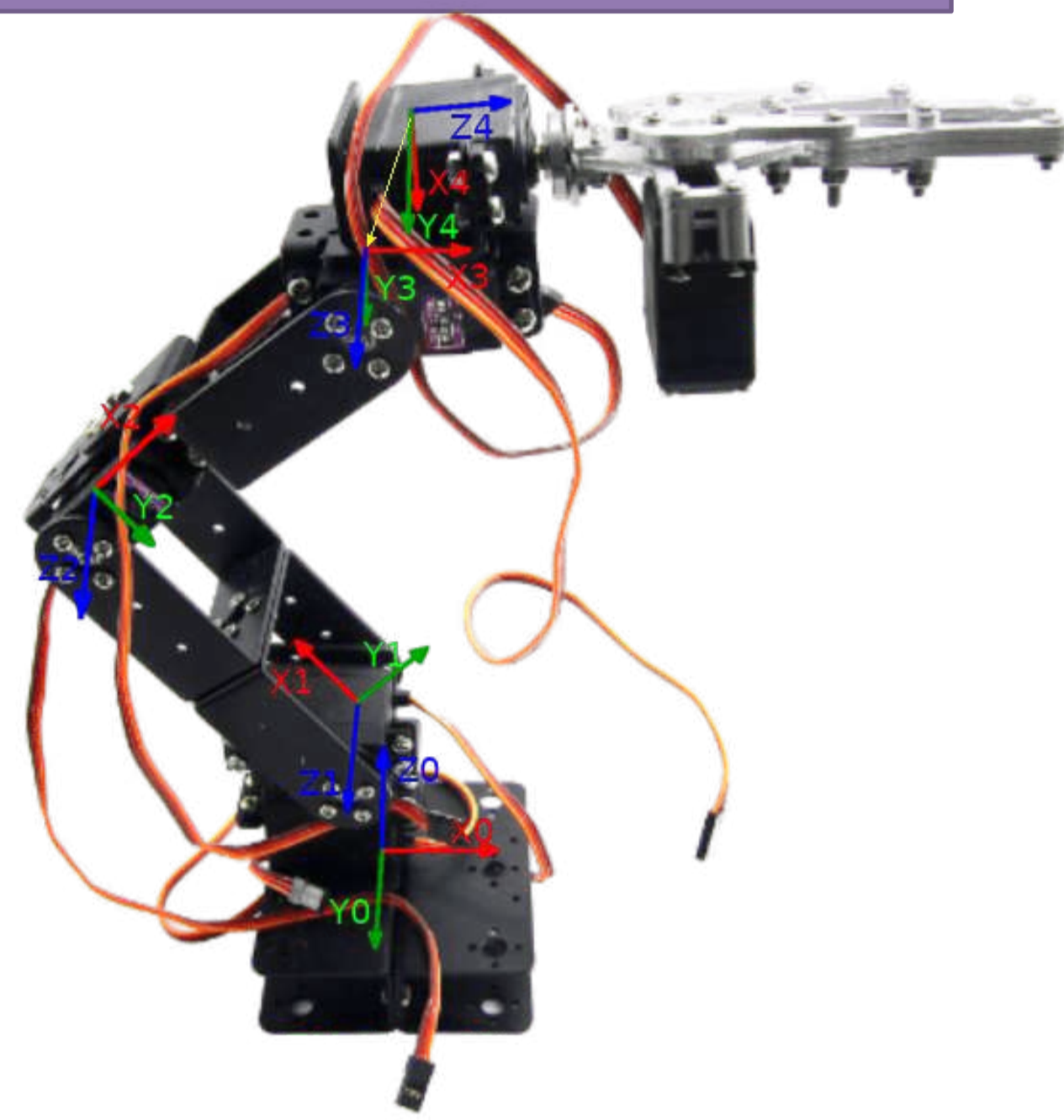


Figure 2. Identification of reference systems in the articulations of the manipulator of five degrees of freedom

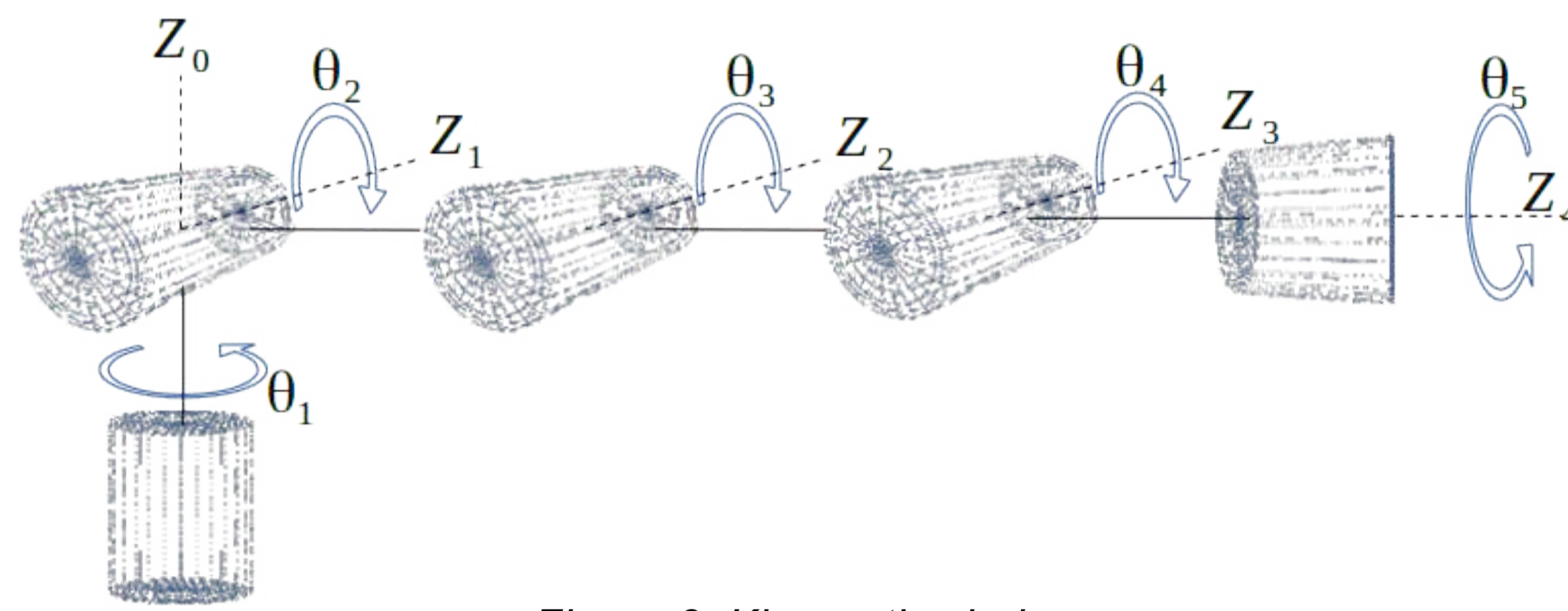


Figure 3. Kinematic chain

### 3.2 Kinematics model

i	$\theta_i$	$\alpha_i$	$a_i$	$d_i$
1	90	-90	0	0
2	0	0	$a_2=10.4\text{ cm}$	0
3	0	0	$a_3=9.8\text{ cm}$	0
4	90	90	0	0
5	0	0	$a_5=6.1\text{ cm}$	$d_5=3\text{ cm}$

Table 1. D-H Parameters

$$\begin{pmatrix} \dot{X}_f \\ \dot{Y}_f \\ \dot{Z}_f \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{pmatrix}$$

3) Jacobian

### 3.3 Dynamic model

$$L(q, \dot{q}) = K(q, \dot{q}) - U(q) \quad \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = \tau$$

3) Lagrangian equation

4) General equation of movement

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$$

5) Handle position torques equation

### 3.4 Block diagram from the dynamic or kinematic model

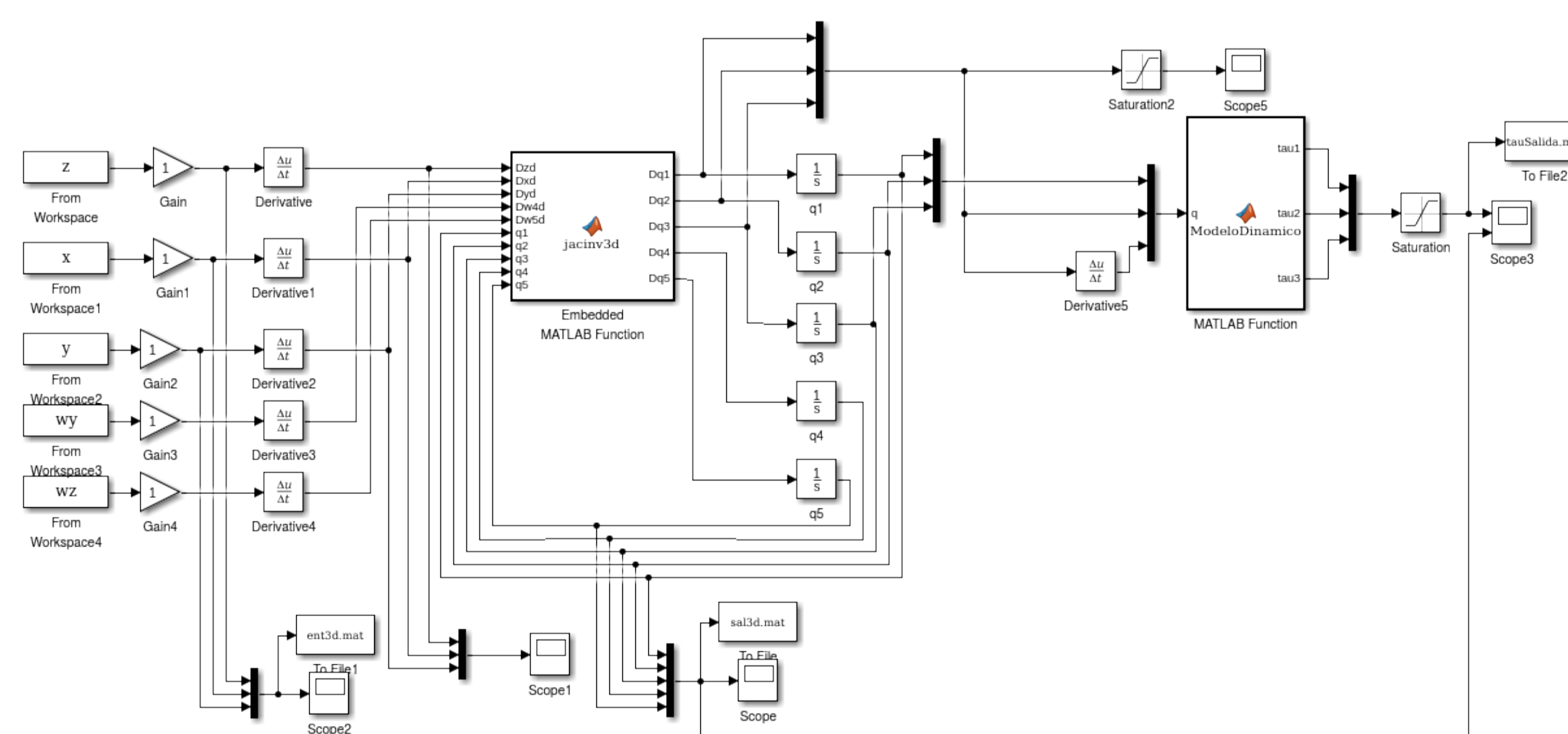


Figure 4. Simulation scheme of the model of kinematics and dynamics of the manipulator.

## 4. Results

Results of experiments obtained in the simulation of mathematical models by kinematics and dynamics, and response of the implementation in the manipulator

### 4.1 3D simulation of kinematic chain

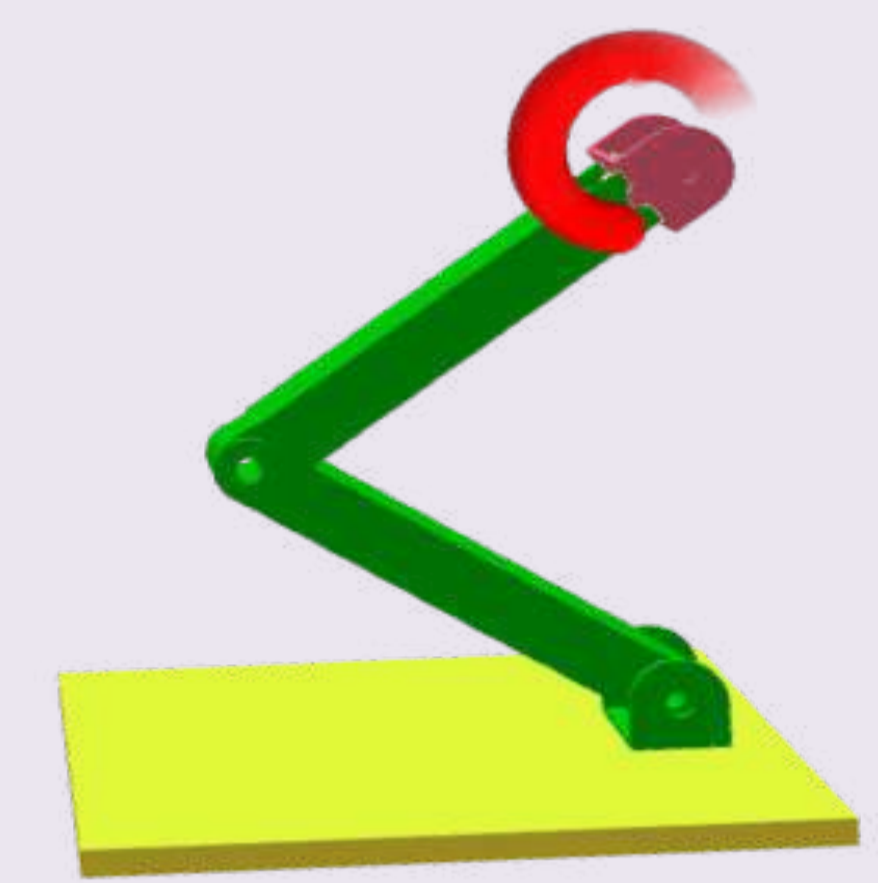


Figure 5. CAD trajectory simulation

### 4.2 Simulation from a kinematic model

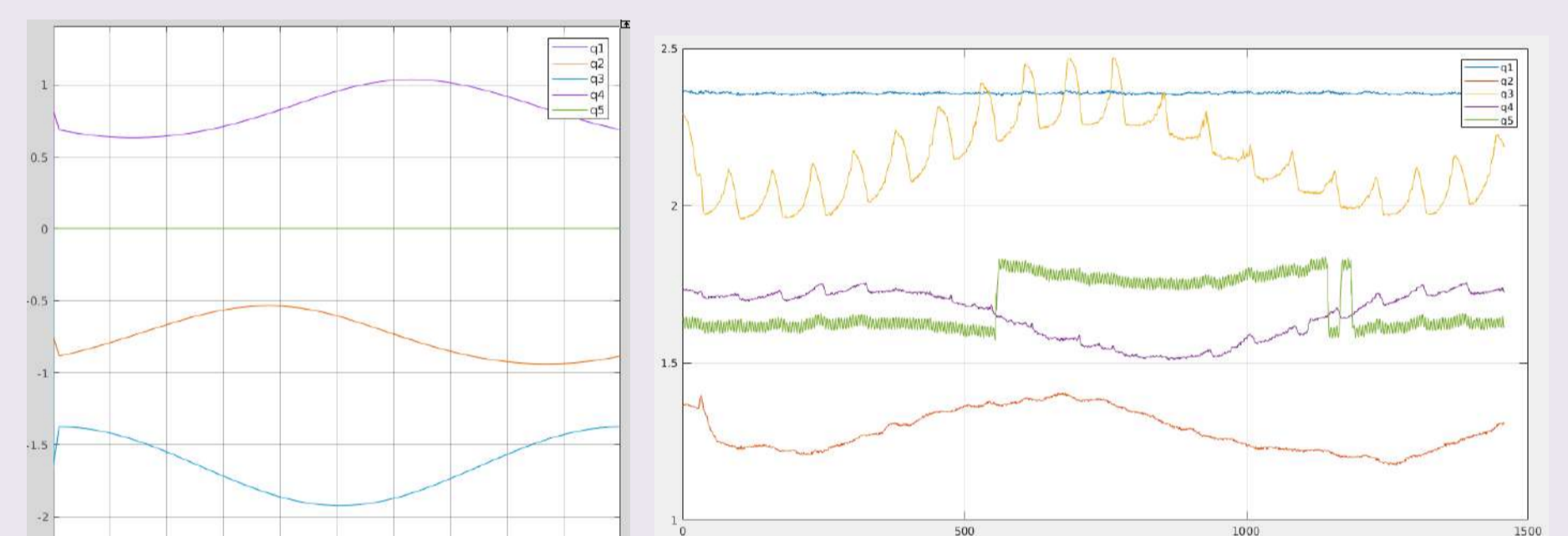


Figure 6. Required response in the articulations of the manipulator to trace trajectory

Figure 8. Response of the position of the articulators of the manipulator to trace trajectory

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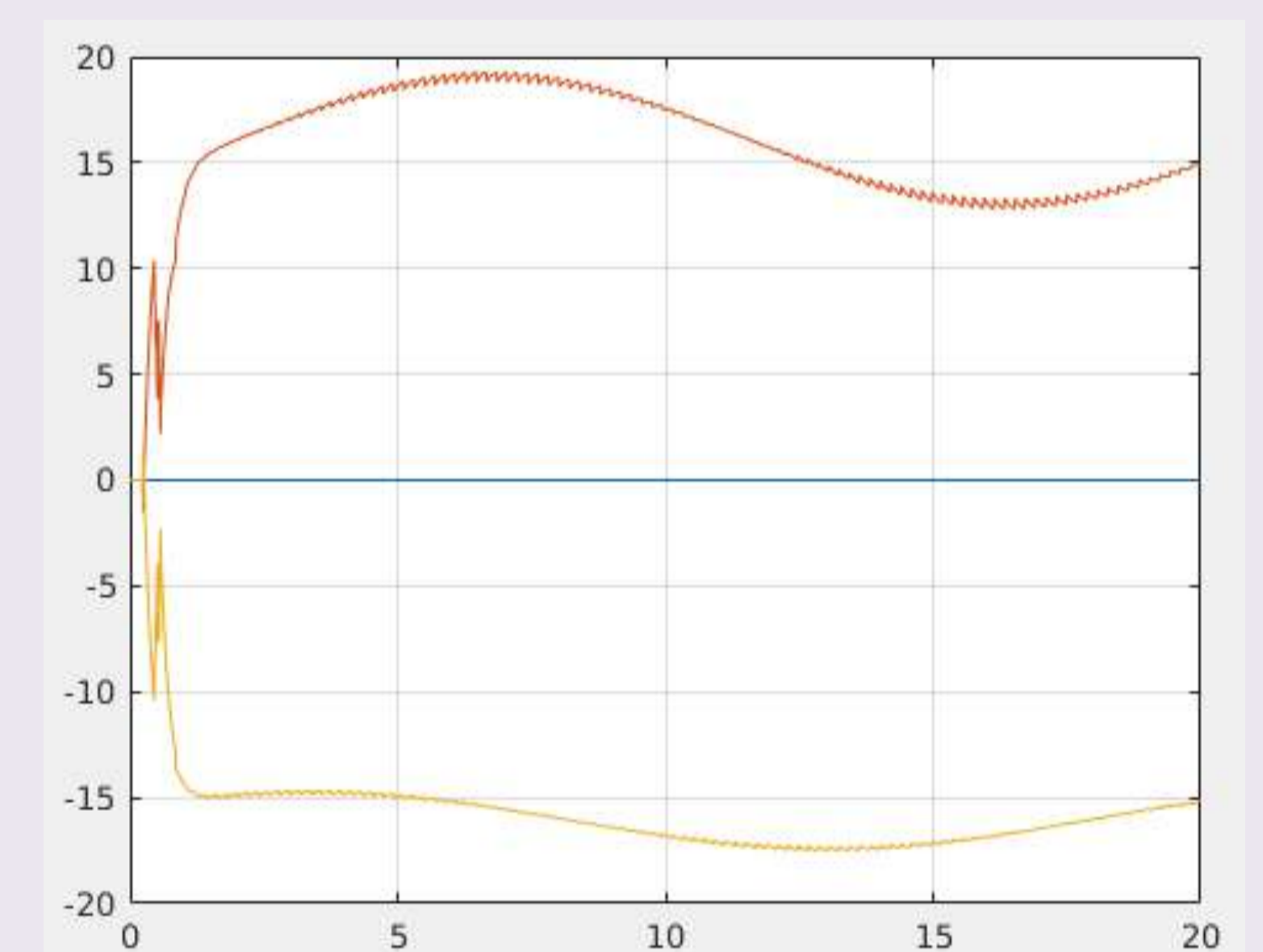


Figure 7. Estimated torque in the actuators of the manipulator to the trace trajectory

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In the construction of the mathematical model of the manipulator arm, kinematic decoupling was considered to obtain the equations that determine the position and orientation of the end effector, the Denavit-Hartenberg method was carried out, finding physical characteristics of the manipulator that facilitate the obtaining of the transformations related to each link with the base reference system. With the Jacobian method we obtain the inverse kinematics of the system, taking into account that the inverse of the matrix that defines it, it can be indeterminate depending on the configuration of the manipulator, so the singularity of said matrix is verified in each point that composes the defined trajectory. The energy of the system, composed of kinetics and potential, is determined to obtain the dynamic model, for this it is considered that the joints in the end effector are fixed, so it can be taken as a rigid part of the last link, because the influence of the orientation is minimal to affect the dynamics with which the torque is estimated in the actuators of the joints. For verification of the model obtained was implemented in the physical system, consecutively the setpoint was given to the manipulator to be placed at each point that forms the path, simultaneously its real position was captured for comparison with the simulated values, resulting in the developed equations defining the system to perform more complex simulations such as the design of strategies or methods to improve the accuracy of the manipulator's movement.

## References

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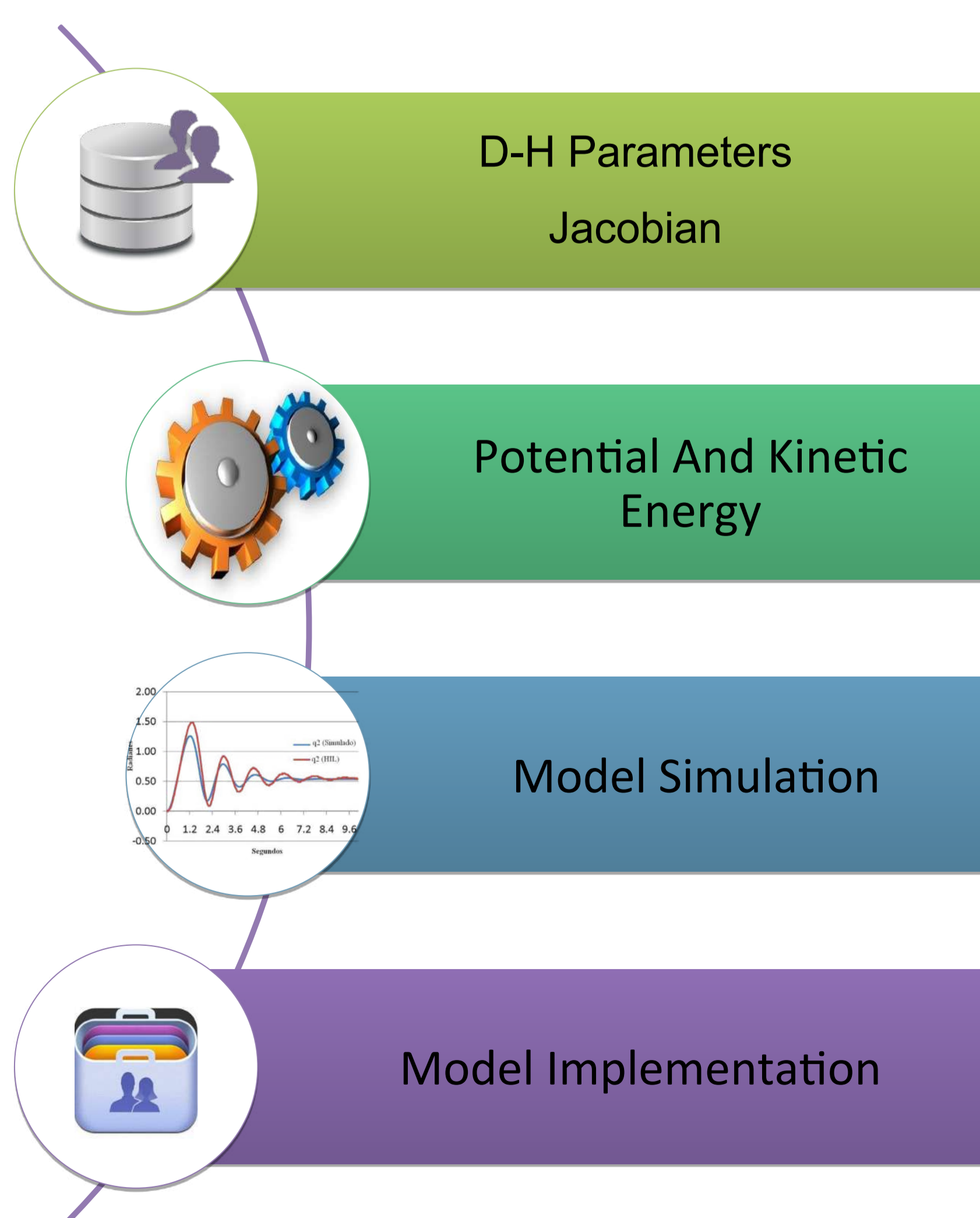


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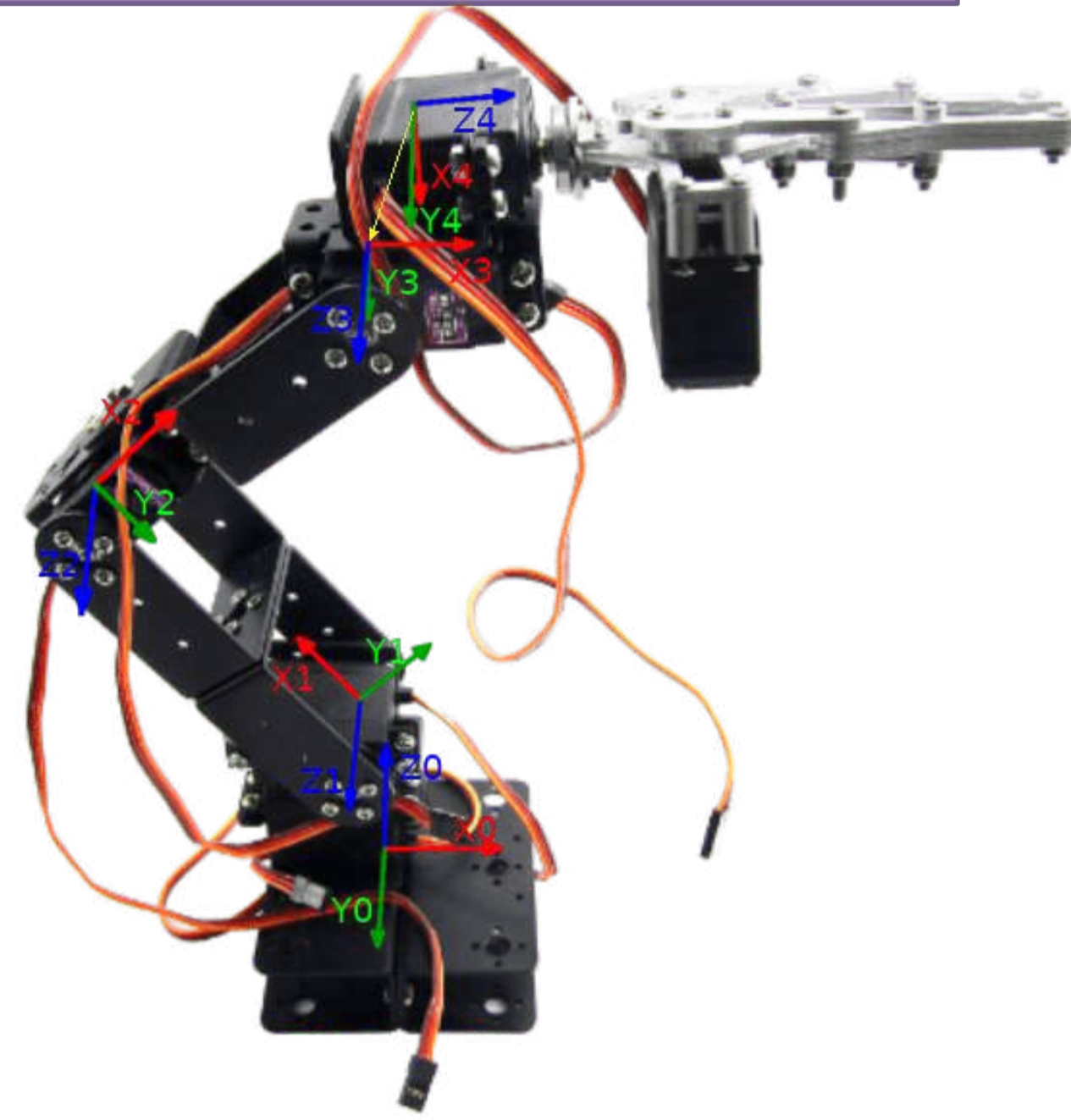


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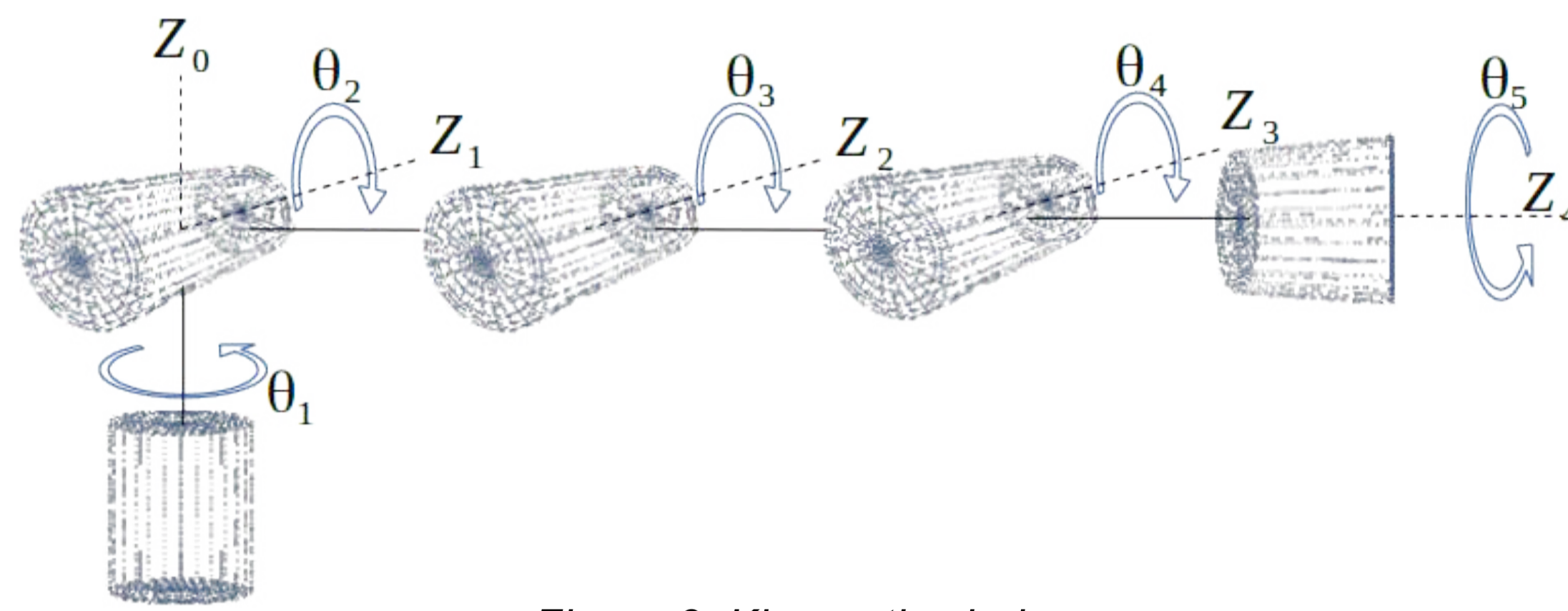


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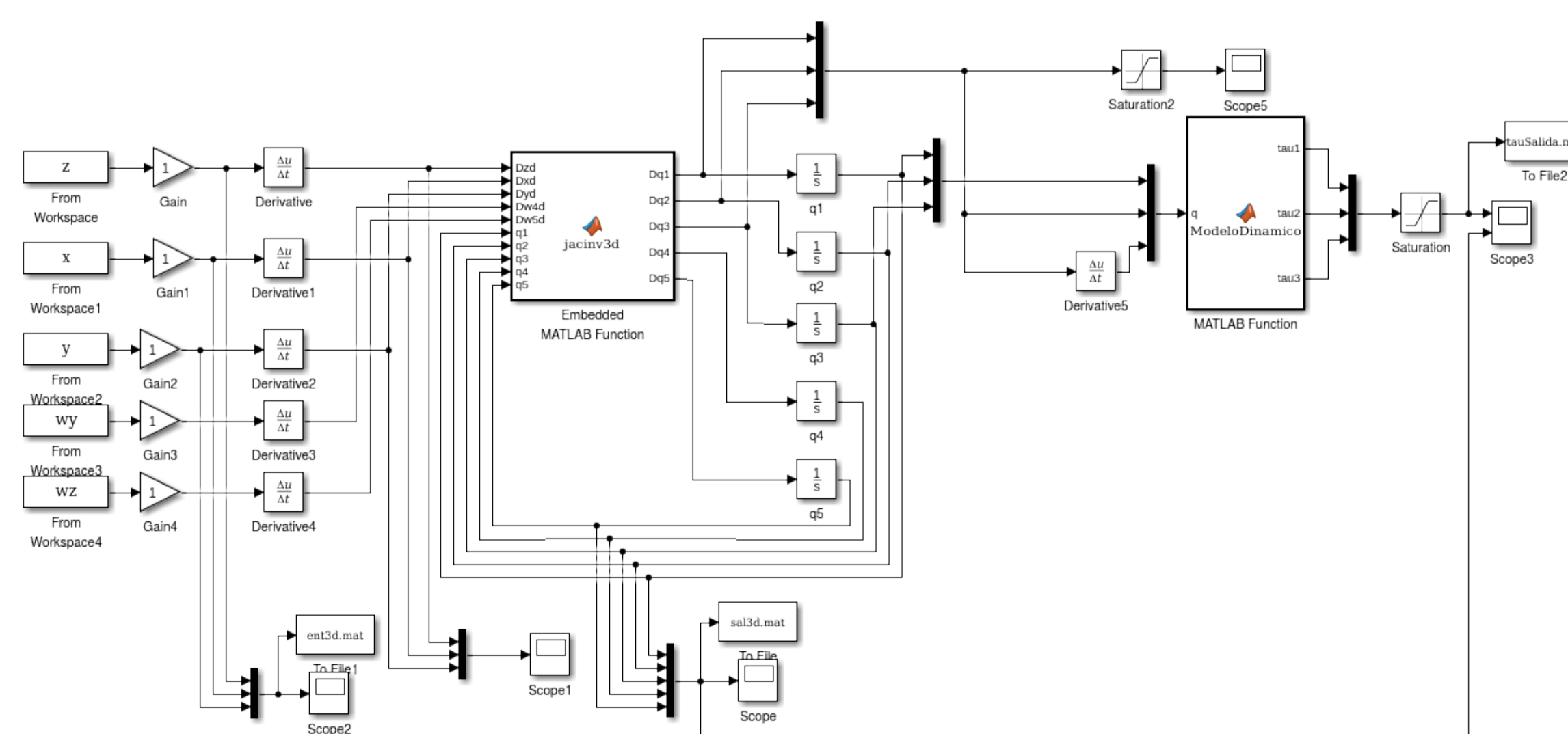


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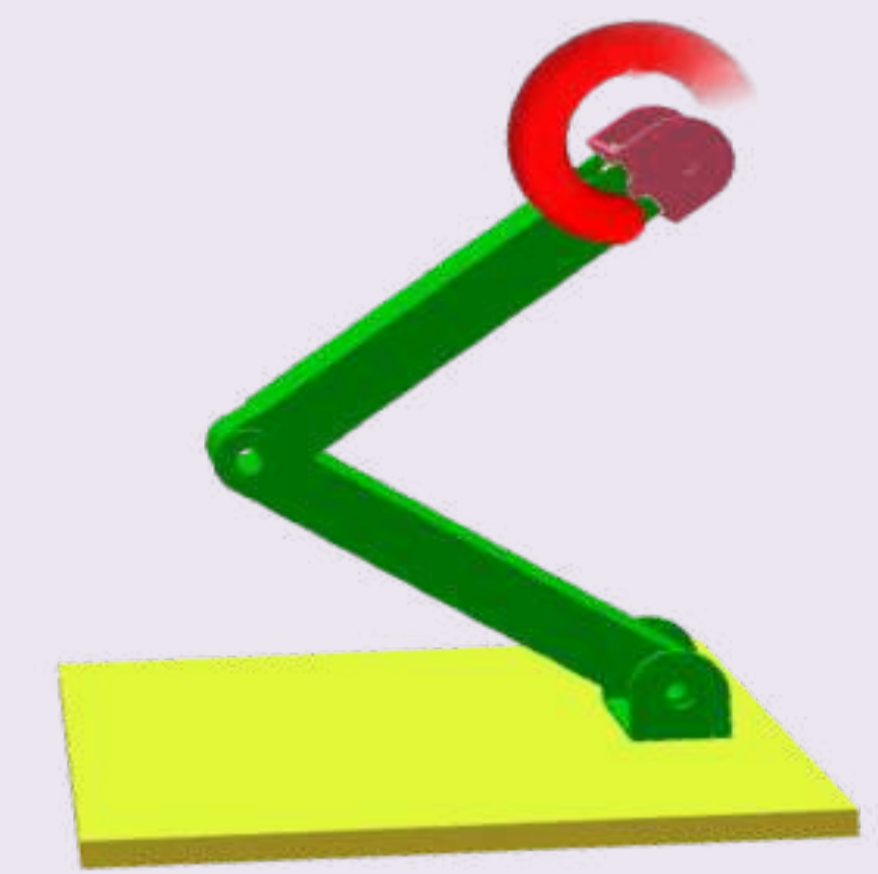


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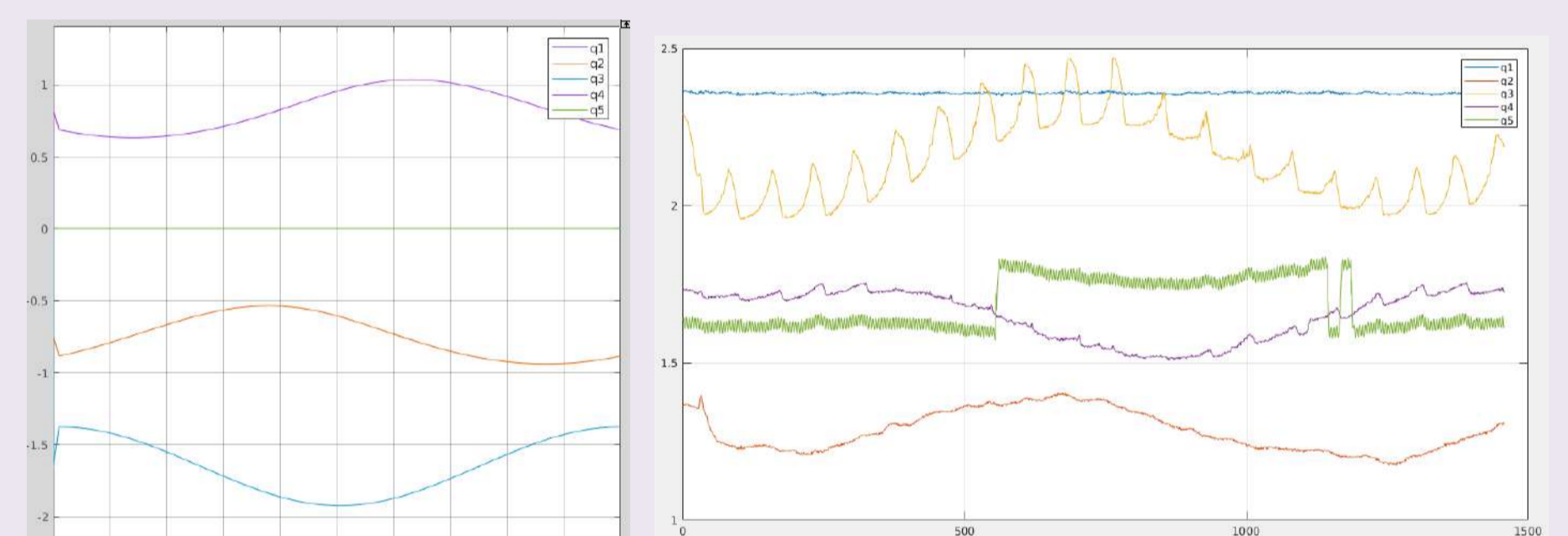


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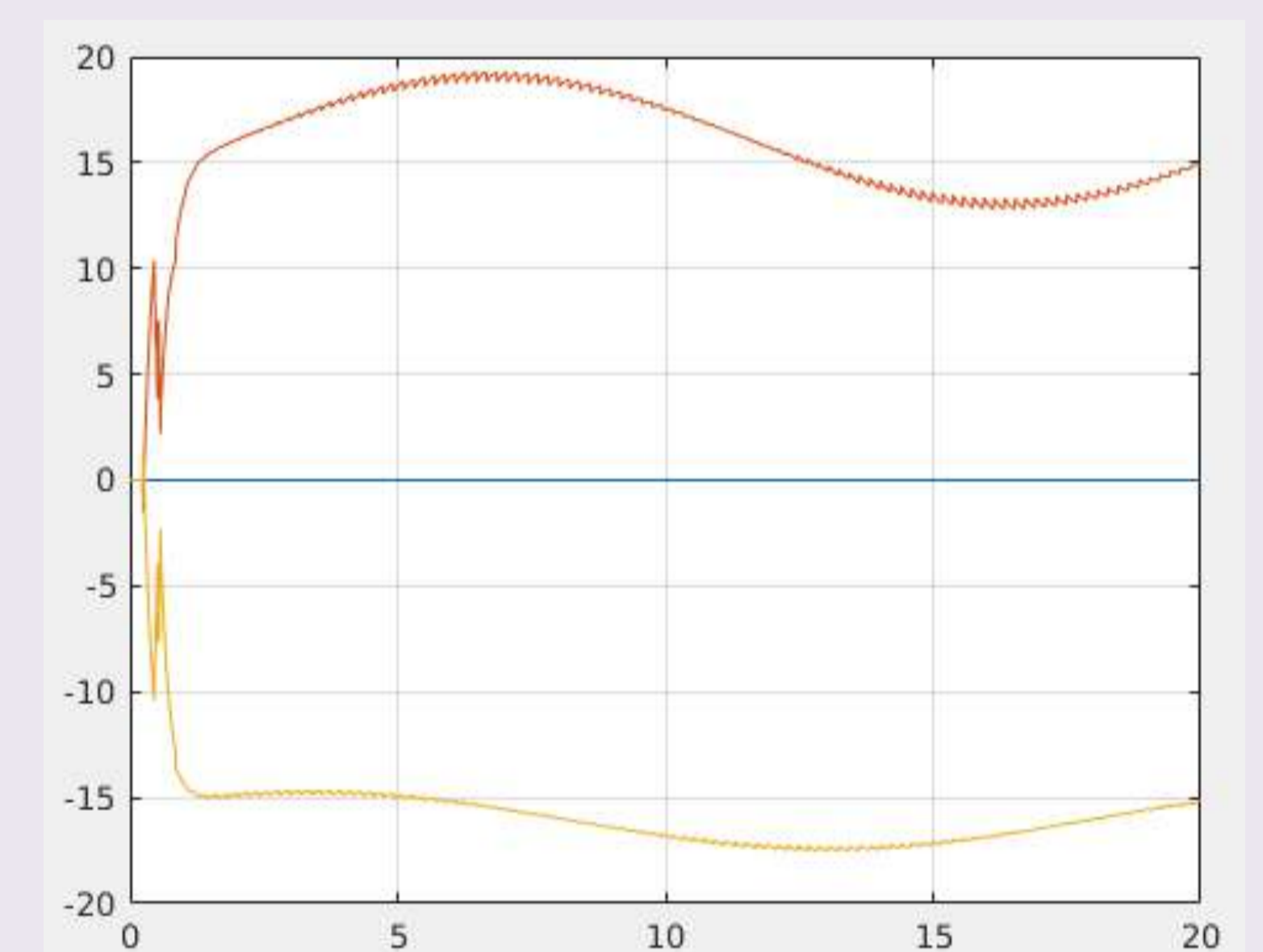


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